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INTERNATIONAL
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Aerodynamics
for Engineers

SIXTH
EDITION

Bertin
Cummings

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INTERNATIONAL
EDITION

Aerodynamics for Engineers

SIXTH EDITION

John J. Bertin • Russell M. Cummings

ALWAYS LEARNING

PEARSON

AERODYNAMICS FOR ENGINEERS

Sixth Edition

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and

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Preface

A great deal has happened since the preface to the fifth edition of *Aerodynamics for Engineers* was written early in 2008. During the spring and early summer of 2008, John Bertin and I were busy checking chapter proofs for "The Book" (as he liked to call it). John was at home in Houston and teaching at his beloved Rice University (you may have noticed that covers of the various editions of *Aerodynamics for Engineers* were usually blue and light gray, the colors of Rice University). I was a visiting researcher at the Institute of Aerodynamics and Flow Technology at The German Aerospace Center (DLR) in Braunschweig. John had two major struggles in his life at the time: he was working through the last stages of the illness that would take his wife, Ruth, from him. He had also been diagnosed with pancreatic cancer, and was dealing with doctors, treatments, and hospitals. We spoke on the phone often about the various challenges he was facing, both with his wife's and his own health. Through the support of his family, as well as his desire to finish the fifth edition, he made it through the summer of 2008 in reasonably good shape. Copies of the book were shipped to us in July 2008, and he was very glad that we had finished the undertaking we had started so many years earlier.

Unfortunately, John's pancreatic cancer took a turn for the worse in late summer of 2008, and he passed away on October 11, 2008. A large number of former co-workers from NASA and various universities, as well as his family and friends, attended his funeral later that month, and we all knew that a very special person had passed from our ranks.

One of the things that John and I talked about during his last months of life was his desire for *Aerodynamics for Engineers* to continue to grow and evolve, even if he was not around to help with that task. I cannot help but think that he asked me to be his co-author for the fifth edition for this purpose. So, in spite of the fact that John is no longer with us, his spirit and excitement for learning will continue to live.

So, there were many goals for writing the sixth edition of *Aerodynamics for Engineers*: (1) to continue the legacy of Professor Bertin; (2) to rewrite many of the sections that provide readers with a motivation for studying aerodynamics in a more casual, enjoyable, and readable manner; (3) to update the technical innovations and advancements that have taken place in aerodynamics since the writing of the previous edition; and (4) to add aerodynamics concept boxes throughout the book to enhance the interest of readers.

To help achieve these goals, I provided readers with new sections, listed under What's New to This Edition on the next page. In addition, there are numerous new figures containing updated information, as well as numerous, additional up-to-date references throughout the book. Finally, numerous new example problems have been added throughout the book to enhance the learning of aerodynamics by the reader, and answers to selected problems have been added to help students know when they have done the problems correctly. Users of the fifth edition of the book will find that all material included in that edition is still included in the sixth edition, with the new material added throughout the book to bring a real-world flavor to the concepts being developed. I hope that readers will find the inclusion of all of this additional material helpful and informative.

Finally, no major revision of a book like *Aerodynamics for Engineers* can take place without the help of many people. I am especially indebted to everyone who aided in collecting new materials for the sixth edition. I want to especially thank Preston A. Henne and Robert van't Riet of McDonnell Douglas; Eli Reshotko of Case Western Reserve University; David W. Hall of DHC Engineering; Stuart Rogers of NASA Ames Research Center; David McDaniel of the University of Alabama, Birmingham; Hans Hornung of Caltech; Andreas Schütte, Thomas Streit, and Martin Hepperle of DLR; Patrick Champigny of ONERA; Aaron Byerley of the U.S. Air Force Academy; John McMasters of The Boeing Company; and William H. Mason of Virginia Tech. In addition, I am very grateful for the excellent suggestions and comments made by the reviewers of the sixth edition: Roger L. Simpson of Virginia Tech, Tej R. Gupta of Embry-Riddle Aeronautical University, Serhat Hosder of Missouri University of Science and Technology, and Lisa Grega of The College of New Jersey. The editorial and production staff at Pearson has been outstanding in their support of this new edition: I greatly appreciate their efforts. I am also extremely grateful to the many students at the U.S. Air Force Academy who have pointed out errors that they found in the previous edition. I hope that everyone who reads this book will find it useful and educational.

The publishers would like to thank Ramesh Kolluru of BMS College of Engineering, Bangalore for reviewing the content of the International Edition.

WHAT'S NEW TO THIS EDITION?

- Aerodynamics concept boxes added throughout the book to bring real-world examples and applications to light as new material is being learned
- Chapter objectives to give readers a better understanding of the goal of each chapter and what concepts they should understand after reading through the chapter
- Significant re-writing of material and derivations from previous editions to improve clarity and usefulness
- Extra example problems to improve understanding of how to apply concepts to useful applications
- Significant new sections added on the topics of: importance of aerodynamics to aircraft performance, a description of the airplane, the irrotational flow condition, applications of potential flow theory to aerodynamics, expanded description of airfoil geometry and nomenclature, high lift military airfoils, the effect of taper ratio on wing efficiency, induced drag estimation, converging-diverging nozzles, shock/shock interactions, subsonic compressible transformations, additional compressibility corrections, critical Mach number, drag divergence Mach number, base drag, and the distinguishing characteristics of hypersonic flow
- Updated figures and photographs to help readers see concepts from real examples and on real aircraft
- Answers to selected problems

Enjoy your study of aerodynamics!

INSTRUCTORS RESOURCES

Resources to accompany the text are located on the Instructor Resource Center website at www.pearsoninternationaleditions.com/cummings. If you are in need of a login and password for this site, please contact your local Pearson representative. Resources include; Instructor Solutions Manual, Matlab files for several example problems and lecture slides for most chapters.

Russell M. Cummings
Larkspur, Colorado

1 WHY STUDY AERODYNAMICS?

Chapter Objectives

- Learn why aerodynamics is important in determining the performance characteristics of airplanes
- Develop a basic understanding of fluid properties such as density, temperature, pressure, and viscosity and know how to calculate these properties for a perfect gas
- Learn about the atmosphere and why we use a "standard atmosphere" model to perform aerodynamic calculations; learn how to perform calculations of fluid properties in the atmosphere
- Learn the basic components of an airplane and what they are used for

The study of aerodynamics is a challenging and rewarding discipline within aeronautics since the ability of an airplane to perform (how high, how fast, and how far an airplane will fly, such as the F-15E shown in Fig. 1.1) is determined largely by the aerodynamics of the vehicle. However, determining the aerodynamics of a vehicle (finding the lift and drag) is one of the most difficult things you will ever do in engineering, requiring complex theories, experiments in wind tunnels, and simulations using modern high-speed computers. Doing any of these things is a challenge, but a challenge well worth the effort for those wanting to better understand aircraft flight.

Figure 1.1 Aerodynamics is required for all components of the F-15E in flight, including the wing, fuselage, horizontal and vertical tails, stores, and how they interact with each other (U.S. Air Force photo by Staff Sgt. Samuel Rogers).

In order to prepare you for the challenge of learning about aerodynamics, we will first look at some interesting aspects of aircraft performance, and how we could determine if one airplane will outperform another airplane in a dog fight. Hopefully this will lead us to the point where we realize that aerodynamics is one of the prime characteristics of an airplane, which will determine the performance of the vehicle.

Of course, aerodynamics also requires that we understand some basic information about fluid dynamics, since physical materials known as fluids are made up of both liquids and gasses, and air is a gas. So some basic concepts about fluid properties and how we can describe a fluid will also be necessary. Since airplanes fly in the atmosphere, we will also develop a standard way to describe the properties of air in the atmosphere. And finally, we will discuss some of the basic geometry of an airplane, so we will have a common nomenclature for discussing how airplanes fly and for the aerodynamics of the various parts of an airplane. All of these pieces of background information will help us get started on the path to understanding aerodynamics, which is the goal of this book.

1.1 AERODYNAMICS AND THE ENERGY-MANEUVERABILITY TECHNIQUE

Early in the First World War, fighter pilots (at least those good enough to survive their first engagement with the enemy) quickly developed tactics that were to serve them throughout the years. German aces, such as Oswald Boelcke and Max Immelman, realized that if they initiated combat starting from an altitude that was greater than that of their adversary, they could dive upon their foe, trading potential energy (height) for kinetic energy (velocity). Using the greater speed of his airplane to close from the rear (i.e., from the target aircraft's "six o'clock position"), the pilot of the attacking aircraft could dictate the conditions of the initial phase of the air-to-air combat. Starting from a superior altitude and converting potential energy to kinetic energy, the attacker might be able to destroy his opponent on the first pass. These tactics were refined, as the successful fighter aces gained a better understanding of the nuances of air combat by building an empirical database through successful air-to-air battles. A language grew up to codify these tactics: "Check your six."

This data base of tactics learned from successful combat provided an empirical understanding of factors that are important to aerial combat. Clearly, the sum of the potential energy plus the kinetic energy (i.e., the total energy) of the aircraft is one of the factors.

EXAMPLE 1.1: The total energy

Compare the total energy of a B-52 (shown in Fig. 1.2a) that weighs 450,000 pounds and that is cruising at a true air speed of 250 knots at an altitude of 20,000 ft with the total energy of an F-5 (shown in Fig. 1.2b) that weighs 12,000 pounds and that is cruising at a true air speed of 250 knots at an altitude of 20,000 ft. The equation for the total energy is

$$E = \frac{1}{2}mV^2 + mgh \tag{1.1}$$

Solution: To have consistent units, the units for velocity should be feet per second rather than knots. A knot is a nautical mile per hour and is equal to 1.69 ft per second, so 250 knots is equal to 422.5 ft/s. The mass is given by the equation:

$$m = \frac{W}{g} \tag{1.2}$$

(a) B-52H (b) F-5E

Figure 1.2 Aircraft used in energy-maneuverability comparison (U.S. Air Force photos; B-52H photo by Mike Cassidy).

Note that the units of mass could be grams, kilograms, lbm, slugs, or $\text{lbf} \cdot \text{s}^2/\text{ft}$. The choice of units often will reflect how mass appears in the application. The mass of the "Buff" (i.e., the B-52) is 13,986 $\text{lbf} \cdot \text{s}^2/\text{ft}$ or 13,986 slugs, while the mass for the F-5 is 373 slugs. The total energy for the B-52 is:

$$E = \frac{1}{2}\left(13{,}986\frac{\text{lbf} \cdot \text{s}^2}{\text{ft}}\right)\left(422.5\frac{\text{ft}}{\text{s}}\right)^2 + (450{,}000\ \text{lbf})(20{,}000\ \text{ft})$$

$$E = 1.0248 \times 10^{10}\ \text{ft} \cdot \text{lbf}$$

Similarly, the total energy of the F-5 fighter is

$$E = \frac{1}{2}\left(373\frac{\text{lbf} \cdot \text{s}^2}{\text{ft}}\right)\left(422.5\frac{\text{ft}}{\text{s}}\right)^2 + (12{,}000\ \text{lbf})(20{,}000\ \text{ft})$$

$$E = 2.7329 \times 10^{8}\ \text{ft} \cdot \text{lbf}$$

The total energy of the B-52 is 37.5 times the total energy of the F-5. Even though the total energy of the B-52 is so very much greater than that for the F-5, it just doesn't seem likely that a B-52 would have a significant advantage in air-to-air combat with an F-5. Notice that the two aircraft are cruising at the same flight conditions (velocity/altitude combination). So in this case the difference in total energy is in direct proportion to the difference in the weights of the two aircraft. Perhaps the specific energy (i.e., the energy per unit weight) is a more realistic parameter when trying to predict which aircraft would have an edge in air-to-air combat.

EXAMPLE 1.2: The energy height

Since the weight specific energy also has units of height, it will be given the symbol H_e and is called the energy height. Dividing the terms in equation (1.1) by the weight of the aircraft ($W = m\,g$)

$$H_e = \frac{E}{W} = \frac{V^2}{2\,g} + h \qquad \textbf{(1.3)}$$

Compare the energy height of a B-52 flying at 250 knots at an altitude of 20,000 ft with that of an F-5 cruising at the same altitude and at the same velocity.

Solution: The energy height of the B-52 is

$$H_e = \frac{1}{2}\frac{\left(422.5\frac{\text{ft}}{\text{s}}\right)^2}{32.174\frac{\text{ft}}{\text{s}^2}} + 20000\ \text{ft}$$

$$H_e = 22774\ \text{ft}$$

Since the F-5 is cruising at the same altitude and at the same true air speed as the B-52, it has the same energy height (i.e., the same weight specific energy).

If we consider only this weight specific energy, the B-52 and the F-5 are equivalent. This is obviously an improvement over the factor of 37.5 that the “Buff” had over the F-5, when the comparison was made based on the total energy. However, the fact that the energy height is the same for these two aircraft indicates that further effort is needed to provide a more realistic comparison for air-to-air combat.

Based on these examples, there must be some additional parameters that are relevant when comparing the one-on-one capabilities of two aircraft in air-to-air combat. Captain Oswald Boelcke developed a series of rules based on his combat experience as a forty-victory ace by October 19, 1916. Boelcke specified seven rules, or “dicta” [Werner (2005)]. The first five, which deal with tactics, are

1. Always try to secure an advantageous position before attacking. Climb before and during the approach in order to surprise the enemy from above, and dive on him swiftly from the rear when the moment to attack is at hand.
2. Try to place yourself between the sun and the enemy. This puts the glare of the sun in the enemy’s eyes and makes it difficult to see you and impossible to shoot with any accuracy.
3. Do not fire the machine guns until the enemy is within range and you have him squarely within your sights.
4. Attack when the enemy least expects it or when he is preoccupied with other duties, such as observation, photography, or bombing.
5. Never turn your back and try to run away from an enemy fighter. If you are surprised by an attack on your tail, turn and face the enemy with your guns.

Although Boelcke’s dicta were to guide fighter pilots for decades to come, they were experienced-based empirical rules. The first dictum deals with your total energy, the sum of the potential energy plus the kinetic energy. We learned from the first two example calculations that predicting the probable victor in one-on-one air-to-air combat is not based on energy alone.

Note that the fifth dictum deals with maneuverability. ***Energy AND Maneuverability!*** The governing equations should include maneuverability as well as the specific energy.

It wasn’t until almost half a century later that a Captain in the U.S. Air Force brought the needed complement of talents to bear on the problem [Coram (2002)]. Captain John R. Boyd was an aggressive and talented fighter pilot who had an insatiable intellectual curiosity for understanding the scientific equations that had to be the basis of the “Boelcke dicta.” John R. Boyd was driven to understand the physics that was the foundation of the tactics that, until that time, had been learned by experience for the fighter pilot lucky enough to survive his early air-to-air encounters with an enemy. In his role as Director of Academics at the U.S. Air Force Fighter Weapons School, it became not only his passion, but his job.

Air combat is a dynamic ballet of move and countermove that occurs over a continuum of time. Therefore, Boyd postulated that perhaps the time derivatives of the energy height are more relevant than the energy height itself. How fast can we, in the target aircraft, with an enemy on our "six." quickly dump energy and allow the foe to pass? Once the enemy has passed, how quickly can we increase our energy height and take the offensive? John R. Boyd taught these tactics in the Fighter Weapons School. Now he became obsessed with the challenge of developing the science of fighter tactics.

1.1.1 Specific Excess Power

If the pilot of the 12,000 lbf F-5 that is flying at a velocity of 250 knots (422.5 ft/s) and at an altitude of 20,000 ft is to gain the upper hand in air-to-air combat, his aircraft must have sufficient power either to out-accelerate or to outclimb his adversary. Consider the case where the F-5 is flying at a constant altitude. If the engine is capable of generating more thrust than the drag acting on the aircraft, the acceleration of the aircraft can be calculated using Newton's Law:

$$\sum F = m\,a$$

which for an aircraft accelerating at a constant altitude becomes

$$T - D = \frac{W}{g}\frac{dV}{dt} \tag{1.4}$$

Multiplying both sides of equation (1.4) by V and dividing by W gives

$$\frac{(T - D)V}{W} = \frac{V}{g}\frac{dV}{dt} \tag{1.5}$$

which is the specific excess power, P_s.

EXAMPLE 1.3: The specific excess power and acceleration

The left-hand side of equation (1.5) is excess power per unit weight, or specific excess power, P_s. Use equation (1.5) to calculate the maximum acceleration for a 12,000-lbf F-5 that is flying at 250 knots (422.5 ft/s) at 20,000 ft.

Solution: Performance charts for an F-5 that is flying at these conditions indicate that it is capable of generating 3550 lbf thrust (T) with the afterburner lit, while the total drag (D) acting on the aircraft is 1750 lbf. Thus, the specific excess power is

$$P_s = \frac{(T - D)V}{W} = \frac{[(3550 - 1750)\ \text{lbf}]\ 422.5\ \text{ft/s}}{12000\ \text{lbf}} = 63.38\ \text{ft/s}$$

Rearranging equation (1.5) to solve for the acceleration gives

$$\frac{dV}{dt} = P_s\frac{g}{V} = (63.38\ \text{ft/s})\frac{32.174\ \text{ft/s}^2}{422.5\ \text{ft/s}} = 4.83\ \text{ft/s}^2$$

1.1.2 Using Specific Excess Power to Change the Energy Height

Taking the derivative with respect to time of the two terms in equation (1.3), we obtain:

$$\frac{dH_e}{dt} = \frac{V}{g}\frac{dV}{dt} + \frac{dh}{dt} \tag{1.6}$$

The first term on the right-hand side of equation (1.6) represents the rate of change of kinetic energy (per unit weight). It is a function of the rate of change of the velocity as seen by the pilot $\left(\frac{dV}{dt}\right)$. The significance of the second term is even less cosmic. It is the rate of change of the potential energy (per unit weight). Note also that $\left(\frac{dh}{dt}\right)$ is the vertical component of the velocity [i.e., the rate of climb (ROC)] as seen by the pilot on his altimeter. Air speed and altitude—these are parameters that fighter pilots can take to heart.

Combining the logic that led us to equations (1.5) and (1.6) leads us to the conclusion that the specific excess power is equal to the time-rate-of-change of the energy height. So,

$$P_s = \frac{(T - D)V}{W} = \frac{dH_e}{dt} = \frac{V}{g}\frac{dV}{dt} + \frac{dh}{dt} \tag{1.7}$$

Given the specific excess power calculated in Example 1.3, we could use equation (1.7) to calculate the maximum rate-of-climb (for a constant velocity) for the 12,000 lbf F-5 as it passes through 20,000 ft at 250 knots.

$$\frac{dh}{dt} = P_s = 63.38\ \text{ft/s} = 3802.8\ \text{ft/min}$$

Clearly, to be able to generate positive values for the terms in equation (1.7), we need an aircraft with excess power (i.e., one for which the thrust exceeds the drag). Weight is another important factor, since the lighter the aircraft, the greater the benefits of the available excess power.

"Boyd, as a combat pilot in Korea and as a tactics instructor at Nellis AFB in the Nevada desert, observed, analyzed, and assimilated the relative energy states of his aircraft and those of his opponent's during air combat engagements. . . . He also noted that, when in a position of advantage, his energy was higher than that of his opponent and that he lost that advantage when he allowed his energy to decay to less than that of his opponent."

"He knew that, when turning from a steady-state flight condition, the airplane under a given power setting would either slow down, lose altitude, or both. The result meant he was losing energy (the drag exceeded the thrust available from the engine). From these observations, he concluded that maneuvering for position was basically an energy problem. Winning required the proper management of energy available at the conditions existing at any point during a combat engagement" [Hillaker (1997)].

In the mid-1960s, Boyd had gathered energy-maneuverability data on all of the fighter aircraft in the U.S. Air Force inventory and on their adversaries. He sought to understand the intricacies of maneuvering flight. What was it about the airplane that would limit or prevent him from making it to do what he wanted it to do?

1.1.3 John R. Boyd Meet Harry Hillaker

The relation between John R. Boyd and Harry Hillaker "dated from an evening in the mid-1960s when a General Dynamics (GD) engineer named Harry Hillaker was sitting in the Officer's Club at Eglin AFB, Florida, having an after dinner drink. Hillaker's host introduced him to a tall, blustery pilot named John R. Boyd, who immediately launched a frontal attack on GD's F-111 fighter. Hillaker was annoyed but bantered back" [Grier (2004)]. Hillaker countered that the F-111 was designated a fighter-bomber.

"A few days later, he (Hillaker) received a call—Boyd had been impressed by Hillaker's grasp of aircraft conceptual design and wanted to know if Hillaker was interested in more organized meetings."

"Thus was born a group that others in the Air Force dubbed the 'fighter mafia.' Their basic belief was that fighters did not need to overwhelm opponents with speed and size. Experience in Vietnam against nimble Soviet-built MiGs had convinced them that technology had not yet turned air-to-air combat into a long-range shoot-out." [Grier (2004)]

The fighter mafia knew that a small aircraft could enjoy a high thrust-to-weight ratio: small aircraft have less drag. "The original F-16 design had about one-third the drag of an F-4 in level flight and one-fifteenth the drag of an F-4 at a high angle-of-attack" [Grier (2004)].

1.1.4 The Importance of Aerodynamics to Aircraft Performance

The importance of the previous discussion is that aircraft performance is largely determined by the aerodynamic characteristics of the airplane (as well as the mass properties and thrust of the airplane). Parameters like lift and drag determine aircraft performance such as energy height. Lift and drag also determine more easy-to-understand parameters like range, rate of climb, and glide ratio (which is exactly the lift/drag ratio of the airplane). Without knowing the aerodynamics of the airplane (as well as the mass properties and thrust), we will not be able to determine how well an airplane will perform. This requires knowing the flow field around the airplane so that the pressures, shear stress, and heating on the surface of the airplane can be determined. That is why the study of aerodynamics is an essential stepping stone to gaining a fuller understanding of how an airplane will perform, and how to improve that performance to achieve flight requirements.

1.2 SOLVING FOR THE AEROTHERMODYNAMIC PARAMETERS

The fundamental problem facing the aerodynamicist is to predict the aerodynamic forces and moments and the heat-transfer rates acting on a vehicle in flight. In order to predict these aerodynamic forces and moments with suitable accuracy, it is necessary to be able to describe the pattern of flow around the vehicle. The resultant flow pattern depends on the geometry of the vehicle, its orientation with respect to the undisturbed free stream, and the altitude and speed at which the vehicle is traveling. In analyzing the various flows that an aerodynamicist may encounter, assumptions about the fluid properties may be introduced. In some applications, the temperature variations are so small that they do not affect the velocity field. In addition, for those applications where the temperature variations have a negligible effect on the flow field, it is often assumed that the density is essentially constant. However, in analyzing high-speed flows, the density variations cannot be neglected. Since density is a function of pressure and temperature, it may be expressed in terms of these two parameters. In fact, for a gas in thermodynamic equilibrium, any thermodynamic property may be expressed as a function of two other independent, thermodynamic properties. Thus, it is possible to formulate the governing equations using the enthalpy and the entropy as the flow properties instead of the pressure and the temperature.

1.2.1 Concept of a Fluid

From the point of view of fluid mechanics, matter can be in one of two states—either solid or fluid. The technical distinction between these two states lies in their response to an applied shear, or tangential, stress. A solid can resist a shear stress by a static deformation; a fluid cannot. A *fluid* is a substance that deforms continuously under the action of shearing forces. An important corollary of this definition is that there can be no shear stresses acting on fluid particles if there is no relative motion within the fluid; that is, such fluid particles are not deformed. Thus, if the fluid particles are at rest or if they are all moving at the same velocity, there are no shear stresses in the fluid. This zero shear stress condition is known as the *hydrostatic stress condition*.

A fluid can be either a liquid or a gas. A liquid is composed of relatively closely packed molecules with strong cohesive forces. As a result, a given mass of liquid will occupy a definite volume of space. If a liquid is poured into a container, it assumes the shape of the container up to the volume it occupies and will form a free surface in a gravitational field if unconfined from above. The upper (or free) surface is planar and perpendicular to the direction of gravity. Gas molecules are widely spaced with relatively small cohesive forces. Therefore, if a gas is placed in a closed container, it will expand until it fills the entire volume of the container. A gas has no definite volume. Thus, if it is unconfined, it forms an atmosphere that is essentially hydrostatic.

1.2.2 Fluid as a Continuum

There are two basic ways to develop equations that describe the motion of a system of fluid particles: we can either define the motion of each and every molecule or define the average behavior of the molecules within a given elemental volume. Our primary concern for problems in this text will not be with the motion of individual molecules, but with the general behavior of the fluid. We are concerned with describing the fluid motion in physical spaces that are very large compared to molecular dimensions (the size of molecules), so our elemental volume will contain a large number of molecules. The fluid in these problems may be considered to be a continuous material whose properties can be determined from a statistical average for the particles in the volume: a macroscopic representation. The assumption of a continuous fluid is valid when the smallest volume of fluid that is of interest contains so many molecules that statistical averages are meaningful. In addition, we will assume that the number of molecules within the volume will remain essentially constant even though there is a continuous flux of molecules through the boundaries. If the elemental volume is too large (as large as the vehicle or body being considered), there could be a noticeable variation in the fluid properties determined statistically at various points in the volume.

For example, the number of molecules in a cubic meter of air at room temperature and at sea-level pressure is approximately 2.5×10^{25}. So, there are 2.5×10^{10} molecules in a cube 0.01 mm on a side. The mean free path of the molecules (the average distance a molecule travels between impacts with other molecules) at sea level is 6.6×10^{-8}m. There are sufficient molecules in this volume for the fluid to be considered a continuum, and the fluid properties can be determined from statistical averages. In contrast, at a very high altitude of 130 km there are only 1.6×10^{17} molecules in a cube 1 m on a side; the mean free path at this altitude is 10.2 m. Therefore, at this altitude the fluid cannot be considered a continuum (this is known as low density of rarefied flow).

A parameter that is commonly used to identify the onset of low-density effects is the Knudsen number, which is the ratio of the mean free path to a characteristic dimension of the body. Although there is no definitive criterion, the continuum flow model starts to break down when the Knudsen number is roughly of the order of 0.1. Because rarefied flows describe a fluid that is not a continuum, different equations would have to be derived than those for a continuum. This book will concentrate, however, on the development of equations for flow in a continuum.

1.2.3 Fluid Properties

By employing the concept of a continuum, we can describe the gross behavior of the fluid motion using certain observable, macroscopic properties. Properties used to describe a general fluid motion include the temperature, the pressure, the density, the viscosity, and the speed of sound.

Temperature. We are all familiar with *temperature* in qualitative terms: an object feels hot (or cold) to the touch. However, because of the difficulty in quantitatively defining the temperature, we typically define situations where there is an equality of temperature. Two bodies have equality of temperature when no change in any observable property occurs when they are in thermal contact. Furthermore, two bodies respectively equal in temperature to a third body must be equal in temperature to each other. Because of this observation, an arbitrary scale of temperature can be defined in terms of a convenient temperature for a standard body (e.g., the freezing point of water).

Pressure. Individual molecules of a fluid continually strike a surface that is placed in the fluid because of the random motion of the molecules due to their thermal

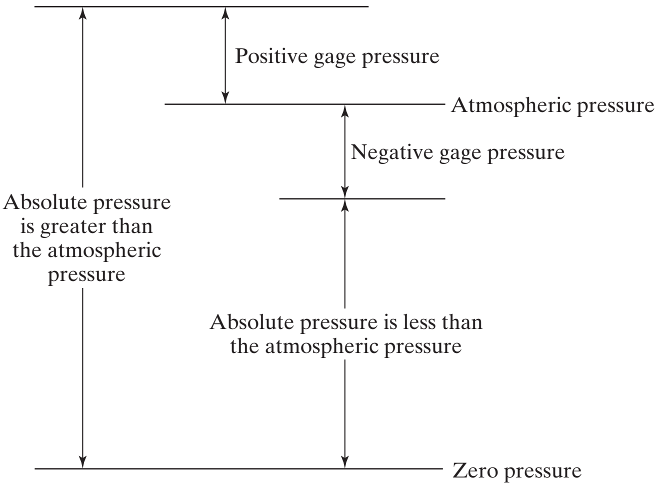

Figure 1.3 Terms used in pressure measurements.

energy. These collisions occur even though the surface is at rest relative to the fluid. By Newton's second law, a force is exerted on the surface equal to the time rate of change of the momentum of the rebounding molecules. *Pressure* is the magnitude of this force per unit area of surface and has units of (force)/(length)^2. Since a fluid that is at rest cannot sustain tangential forces, the pressure on the surface must act in the direction perpendicular to that surface. Furthermore, the pressure acting at a point in a fluid at rest is the same in all directions.

Standard atmospheric pressure at sea level is defined as the pressure that can support a column of mercury 760 mm in length when the density of the mercury is 13.5951 g/cm^3 and the acceleration due to gravity is the standard sea level value. The standard atmospheric pressure at sea level in SI (System International) units is 1.01325×10^5 N/m^2. In English units, the standard atmospheric pressure at sea level is 14.696 lbf/in^2 or 2116.22 lbf/ft^2.

In many aerodynamic applications, we are interested in the difference between the absolute value of the local pressure and the atmospheric pressure. Many pressure gages indicate the difference between the absolute pressure and the atmospheric pressure existing at the gage. This difference, which is referred to as *gage pressure*, is illustrated in Fig. 1.3.

Aerodynamics Concept Box: Consistent Units

Performing calculations with the correct units can be one of the most challenging aspects of aerodynamics (or any field of engineering for that matter). The fact that aerodynamics is often done in both SI and English units can make the challenge even greater. Performing calculations in consistent units can greatly reduce the chance of making errors and greatly increase the ease of getting results.

Consistent units are the units of length, time, force, and mass that make Newton's second law balance for unit values of each term:

$$F = ma$$

$$1 \text{ force unit} = 1 \text{ mass unit} \times 1 \text{ acceleration unit}$$

For calculations in the SI system, the consistent units are Newtons (force), kilograms (mass), and $\mathrm{m/s^2}$ (acceleration), or in other words:

$$1 \text{ Newton} = 1 \text{ kilogram} \times 1 \text{ m/s}^2$$

Using these units in calculations will ensure that units cancel correctly. If any other units appear in a problem, they should immediately be converted to these consistent units prior to performing calculations (grams should be converted to kilograms, centimeters should be converted to meters, hours should be converted to seconds, etc.).

For calculations in the English system, the consistent units are pounds force (often just called pounds), slugs, feet, and seconds:

$$1 \text{ lbf (or lb)} = 1 \text{ slug} \times 1 \text{ ft/s}^2$$

Any other units appearing in problems are most easily dealt with by converting to these consistent units (pounds mass [lbm] should be converted to slugs, miles should be converted to feet, hours should be converted to seconds, etc.). Using consistent units consistently will save a great deal of pain and suffering while performing calculations!

Density. The *density* of a fluid at a point in space is the mass of the fluid per unit volume surrounding the point. As is the case when evaluating the other fluid properties, the incremental volume must be large compared to molecular dimensions yet very small relative to the dimensions of the vehicle whose flow field we seek to analyze. Provided that the fluid may be assumed to be a continuum, the density at a point is defined as

$$\rho = \lim_{\delta(\text{vol}) \to 0} \frac{\delta(\text{mass})}{\delta(\text{vol})} \tag{1.8}$$

where δ represents a change rather than a differential. The dimensions of density are (mass)/(length)^3.

In general, the density of a gas is a function of the composition of the gas, its temperature, and its pressure. The relation

$$\rho(\text{composition}, T, p) \tag{1.9}$$

is known as an *equation of state*. For a thermally perfect gas, the equation of state is

$$\rho = \frac{p}{RT} \tag{1.10}$$

The gas constant R has a particular value for each substance. The gas constant for air has the value 287.05 $\mathrm{N \cdot m/kg \cdot K}$ in SI units and 53.34 $\mathrm{ft \cdot lbf/lbm \cdot {}^\circ R}$ or 1716.16 $\mathrm{ft^2/s^2 \cdot {}^\circ R}$ in English units. The temperature in equation (1.10) should be in absolute units. Thus, the temperature is either in K or in °R, but never in °C or in °F. The density of air at standard day sea level conditions is 1.2250 $\mathrm{kg/m^3}$ or 0.002377 $\mathrm{slug/ft^3}$.

EXAMPLE 1.4: Density in SI units

Calculate the density of air when the pressure is $1.01325 \times 10^5\ \mathrm{N/m^2}$ and the temperature is 288.15 K. Since air at this pressure and temperature behaves as a perfect gas, we can use equation (1.10).

Solution:

$$\rho = \frac{1.01325 \times 10^5\ \mathrm{N/m^2}}{(287.05\ \mathrm{N \cdot m/kg \cdot K})(288.15\ \mathrm{K})}$$
$$= 1.2250\ \mathrm{kg/m^3}$$

EXAMPLE 1.5: Density in English units

Calculate the density of air when the pressure is $2116.22\ \mathrm{lbf/ft^2}$ and the temperature is 518.67°R. Since air at this pressure and temperature behaves as a perfect gas, we can use equation (1.10). Note that throughout the remainder of this book, air will be assumed to behave as a perfect gas unless specifically stated otherwise.

Solution:

$$\rho = \frac{2116.22 \dfrac{\mathrm{lbf}}{\mathrm{ft^2}}}{\left(53.34 \dfrac{\mathrm{ft \cdot lbf}}{\mathrm{lbm \cdot {}^\circ R}}\right)(518.67^\circ\mathrm{R})} = 0.07649 \frac{\mathrm{lbm}}{\mathrm{ft^3}}$$

Alternatively,

$$\rho = \frac{2116.22 \dfrac{\mathrm{lbf}}{\mathrm{ft^2}}}{\left(1716.16 \dfrac{\mathrm{ft^2}}{\mathrm{s^2 \cdot {}^\circ R}}\right)(518.67^\circ\mathrm{R})} = 0.002377 \frac{\mathrm{lbf \cdot s^2}}{\mathrm{ft^4}}$$

The unit $\mathrm{lbf \cdot s^2/ft^4}$ is often written as $\mathrm{slugs/ft^3}$, where slugs are alternative units of mass in the English system. One slug is the equivalent of 32.174 lbm.

For vehicles that are flying at approximately 100 m/s (330 ft/s), or less, the density of the air flowing past the vehicle is assumed constant when obtaining a solution for the flow field. Rigorous application of equation (1.10) would require that the pressure and the temperature remain constant (or change proportionally) in order for the density to remain constant throughout the flow field. We know that the pressure around the vehicle is not constant, since the aerodynamic forces and moments in which we are interested are the result of pressure variations associated with the flow pattern. However, the assumption of constant density for velocities below 100 m/s is a valid approximation because the pressure changes that occur from one point to another in the flow field are small relative to the absolute value of the pressure.

Viscosity. In all real fluids, a shearing deformation is accompanied by a shearing stress. The fluids of interest in this text are *Newtonian* in nature; that is, the shearing stress is proportional to the rate of shearing deformation. The constant of proportionality is called the *coefficient of viscosity*, μ. Therefore,

$$\text{shear stress} = \mu \times \text{transverse gradient of velocity} \tag{1.11}$$

There are many problems of interest for which the effects of viscosity can be neglected. In such problems, the magnitude of the coefficient of viscosity of the fluid and of the velocity gradients in the flow field are such that their product is negligible relative to the inertia of the fluid particles and to the pressure forces acting on them. We will use the term *inviscid flow* in these cases to emphasize the fact that it is the character both of the flow field and of the fluid that allows us to neglect viscous effects. No real fluid has a zero coefficient of viscosity, but there are times when the effects of viscosity are negligible.

The viscosity of a fluid relates to the transport of momentum in the direction of the velocity gradient (but opposite in sense). Therefore, viscosity is a transport property. In general, the coefficient of viscosity is a function of the composition of the gas, its temperature, and its pressure. The viscosity of air is independent of pressure for temperatures below 3000 K (5400°R). In this temperature range, we could use Sutherland's equation to calculate the coefficient of viscosity:

$$\mu = C_1 \frac{T^{1.5}}{T + C_2} \tag{1.12}$$

For SI units where temperature, T, is in units of K and μ is in units of kg/s · m use $C_1 = 1.458 \times 10^{-6}$ and $C_2 = 110.4$. For English units where temperature, T, is in units of °R and μ is in units of lbf · s/ft^2, use $C_1 = 2.27 \times 10^{-8}$ and $C_2 = 198.6$.

EXAMPLE 1.6: Viscosity in SI units

Calculate the viscosity of air when the temperature is 288.15 K.

Solution:

$$\begin{aligned}\mu &= 1.458 \times 10^{-6} \frac{(288.15)^{1.5}}{288.15 + 110.4} \\ &= 1.7894 \times 10^{-5}\ \text{kg/s} \cdot \text{m}\end{aligned}$$

EXAMPLE 1.7: Viscosity in English units

Calculate the viscosity of air when the temperature is 59.0°F.

Solution: First, convert the temperature to the absolute scale for English units, °R, 59.0°F + 459.67 = 518.67°R.

$$\mu = 2.27 \times 10^{-8} \frac{(518.67)^{1.5}}{518.67 + 198.6}$$
$$= 3.7383 \times 10^{-7} \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$$

Equations used to calculate the coefficient of viscosity depend on the model used to describe the intermolecular forces of the gas molecules, so that it is necessary to define the potential energy of the interaction of the colliding molecules. Svehla (1962) noted that the potential energy for the Sutherland model is described physically as a rigid, impenetrable sphere, surrounded by an inverse-power attractive force. This model is qualitatively correct in that the molecules attract one another when they are far apart and exert strong repulsive forces upon one another when they are close together.

Chapman and Cowling (1960) note that equation (1.12) closely represents the variation of μ with temperature over a "fairly" wide range of temperatures. They caution, however, that the success of Sutherland's equation in representing the variation of μ with temperature for several gases does not establish the validity of Sutherland's molecular model for those gases. "In general it is not adequate to represent the core of a molecule as a rigid sphere, or to take molecular attractions into account to a first order only. The greater rapidity of the experimental increase of μ with T, as compared with that for nonattracting rigid spheres, has to be explained as due partly to the 'softness' of the repulsive field at small distances, and partly to attractive forces which have more than a first-order effect. The chief value of Sutherland's formula seems to be as a simple interpolation formula over restricted ranges of temperature" [Chapman and Cowling (1960)].

The Lennard-Jones model for the potential energy of an interaction, which takes into account both the softness of the molecules and their mutual attraction at large distances, has been used by Svehla (1962) to calculate the viscosity and the thermal conductivity of gases at high temperatures. The coefficients of viscosity for air as tabulated by Svehla are compared with the values calculated using equation (1.12) in Table 1.1. These comments are made to emphasize the fact that even the basic fluid properties may involve approximate models that have a limited range of applicability.

Kinematic Viscosity. The aerodynamicist may encounter many applications where the ratio μ/ρ has been replaced by a single parameter. Because this ratio appears frequently, it has been given a special name, the kinematic viscosity. The symbol used to represent the kinematic viscosity is ν, where:

$$\nu = \frac{\mu}{\rho} \tag{1.13}$$

In this ratio, the force units (or, equivalently, the mass units) cancel. Therefore, ν has the dimensions of $(\text{length})^2/(\text{time})$ (e.g., square meters per second or square feet per second).

TABLE 1.1 Comparison of the Coefficient of Viscosity for Air as Tabulated by Svehla (1962) and as Calculated Using Sutherland's Equation [Equation (1.12)]

T (K)	$\mu \times 10^5$ (kg/m·s)*	$\mu \times 10^5$ (kg/m·s)†
200	1.360	1.329
400	2.272	2.285
600	2.992	3.016
800	3.614	3.624
1000	4.171	4.152
1200	4.695	4.625
1400	5.197	5.057
1600	5.670	5.456
1800	6.121	5.828
2000	6.553	6.179
2200	6.970	6.512
2400	7.373	6.829
2600	7.765	7.132
2800	8.145	7.422
3000	8.516	7.702
3200	8.878	7.973
3400	9.232	8.234
3600	9.579	8.488
3800	9.918	8.734
4000	10.252	8.974
4200	10.580	9.207
4400	10.902	9.435
4600	11.219	9.657
4800	11.531	9.874
5000	11.838	10.087

* From Svehla (1962)

† Calculated using equation (1.12)

EXAMPLE 1.8: Kinematic Viscosity in English units

Using the results of Examples 1.5 and 1.7, calculate the kinematic viscosity of air when the temperature is 518.67°R and the pressure is 2116.22 lbf/ft^2.

Solution: From Example 1.5, $\rho = 0.07649\,\text{lbm/ft}^3 = 0.002377\,\text{lbf}\cdot\text{s}^2/\text{ft}^4$; while from Example 1.7, $\mu = 3.7383 \times 10^{-7}\,\text{lbf}\cdot\text{s/ft}^2$. Therefore,

$$\nu = \frac{\mu}{\rho} = \frac{3.7383 \times 10^{-7}\dfrac{\text{lbf}\cdot\text{s}}{\text{ft}^2}}{0.002377\dfrac{\text{lbf}\cdot\text{s}^2}{\text{ft}^4}} = 1.573 \times 10^{-4}\frac{\text{ft}^2}{\text{s}}$$

If we use the alternative units for the density, we must employ the factor g_c, which is equal to $32.174\,\text{ft}\cdot\text{lbm/lbf}\cdot\text{s}^2$, to arrive at the appropriate units.

$$\nu = \frac{\mu}{\rho} = \frac{3.7383 \times 10^{-7}\dfrac{\text{lbf}\cdot\text{s}}{\text{ft}^2}}{0.07649\dfrac{\text{lbm}}{\text{ft}^3}}\left(32.174\frac{\text{ft}\cdot\text{lbm}}{\text{lbf}\cdot\text{s}^2}\right)$$

$$= 1.573 \times 10^{-4}\,\text{ft}^2/\text{s}$$

Speed of Sound. The speed at which a disturbance of infinitesimal proportions propagates through a fluid that is at rest is known as the *speed of sound*, which is designated in this book as a (the acoustic speed). The speed of sound is established by the properties of the fluid. For a perfect gas $a = \sqrt{\gamma RT}$, where γ is the ratio of specific heats (see Chapter 8) and R is the gas constant. For the range of temperature over which air behaves as a perfect gas, $\gamma = 1.4$ and the speed of sound is given by

$$a = 20.047\sqrt{T} \tag{1.14a}$$

where T is the temperature in K and the units for the speed of sound are m/s. In English units

$$a = 49.02\sqrt{T} \tag{1.14b}$$

where T is the temperature in °R and the units for the speed of sound are ft/s.

1.2.4 Pressure Variation in a Static Fluid Medium

In order to compute the forces and moments or the heat-transfer rates acting on a vehicle, or to determine the flight path (i.e., the trajectory) of the vehicle, we will often need an analytic model of the atmosphere instead of using a table, such as Table 1.2. To do this, we will develop the equations describing the pressure variation in a static fluid medium. If fluid particles, when viewed as a continuum, are either all at rest or all moving with the same velocity, the fluid is said to be a *static medium*. Thus, the term *static fluid properties* may be applied to situations in which the elements of the fluid are moving, provided that there is no relative motion between fluid elements. Since there is no relative motion between adjacent layers of the fluid, there are no shear forces. So, with no relative motion between fluid elements, the viscosity of the fluid is of no concern. For these inviscid flows, the only forces acting on the surface of the fluid element are pressure forces.

TABLE 1.2A U.S. Standard Atmosphere, 1976 SI Units

Geometric Altitude (*km*)	*Pressure* (N/m^2)	*Temperature* (K)	*Density* (kg/m^3)	*Viscosity* ($kg/m \cdot s$)	*Speed of Sound* (m/s)
0	1.0133 *E* + 05	288.150	1.2250 *E* + 00	1.7894 *E* − 05	340.29
1	8.9875 *E* + 04	281.651	1.1117 *E* + 00	1.7579 *E* − 05	336.43
2	7.9501 *E* + 04	275.154	1.0066 *E* + 00	1.7260 *E* − 05	332.53
3	7.0121 *E* + 04	268.659	9.0926 *E* − 01	1.6938 *E* − 05	328.58
4	6.1669 *E* + 04	262.166	8.1934 *E* − 01	1.6612 *E* − 05	324.59
5	5.4048 *E* + 04	255.676	7.3643 *E* − 01	1.7885 *E* − 05	320.55
6	4.7217 *E* + 04	249.187	6.6012 *E* − 01	1.5949 *E* − 05	316.45
7	4.1105 *E* + 04	242.700	5.9002 *E* − 01	1.5612 *E* − 05	312.31
8	3.5651 *E* + 04	236.215	5.2578 *E* − 01	1.5271 *E* − 05	308.11
9	3.0800 *E* + 04	229.733	4.6707 *E* − 01	1.4926 *E* − 05	303.85
10	2.6500 *E* + 04	223.252	4.1351 *E* − 01	1.4577 *E* − 05	299.53
11	2.2700 *E* + 04	216.774	3.6481 *E* − 01	1.4223 *E* − 05	295.15
12	1.9399 *E* + 04	216.650	3.1193 *E* − 01	1.4216 *E* − 05	295.07
13	1.6579 *E* + 04	216.650	2.6660 *E* − 01	1.4216 *E* − 05	295.07
14	1.4170 *E* + 04	216.650	2.2786 *E* − 01	1.4216 *E* − 05	295.07
15	1.2111 *E* + 04	216.650	1.9475 *E* − 01	1.4216 *E* − 05	295.07
16	1.0352 *E* + 04	216.650	1.6647 *E* − 01	1.4216 *E* − 05	295.07
17	8.8497 *E* + 03	216.650	1.4230 *E* − 01	1.4216 *E* − 05	295.07
18	7.5652 *E* + 03	216.650	1.2165 *E* − 01	1.4216 *E* − 05	295.07
19	6.4675 *E* + 03	216.650	1.0400 *E* − 01	1.4216 *E* − 05	295.07
20	5.5293 *E* + 03	216.650	8.8911 *E* − 02	1.4216 *E* − 05	295.07
21	4.7289 *E* + 03	217.581	7.5715 *E* − 02	1.4267 *E* − 05	295.70
22	4.0474 *E* + 03	218.574	6.4510 *E* − 02	1.4322 *E* − 05	296.38
23	3.4668 *E* + 03	219.567	5.5006 *E* − 02	1.4376 *E* − 05	297.05
24	2.9717 *E* + 03	220.560	4.6938 *E* − 02	1.4430 *E* − 05	297.72
25	2.5491 *E* + 03	221.552	4.0084 *E* − 02	1.4484 *E* − 05	298.39
26	2.1883 *E* + 03	222.544	3.4257 *E* − 02	1.4538 *E* − 05	299.06
27	1.8799 *E* + 03	223.536	2.9298 *E* − 02	1.4592 *E* − 05	299.72
28	1.6161 *E* + 03	224.527	2.5076 *E* − 02	1.4646 *E* − 05	300.39
29	1.3904 *E* + 03	225.518	2.1478 *E* − 02	1.4699 *E* − 05	301.05
30	1.1970 *E* + 03	226.509	1.8411 *E* − 02	1.4753 *E* − 05	301.71

TABLE 1.2B U.S. Standard Atmosphere, 1976 English Units

Geometric Altitude (kft)	*Pressure (lbf/ft^2)*	*Temperature (°R)*	*Density (slug/ft^3)*	*Viscosity (slug/ft · s)*	*Speed of Sound (ft/s)*
0	2.1162 *E* + 03	518.67	2.3769 *E* − 03	3.7383 *E* − 07	1116.44
2	1.9677 *E* + 03	511.54	2.2409 *E* − 03	3.6982 *E* − 07	1108.76
4	1.8277 *E* + 03	504.41	2.1109 *E* − 03	3.6579 *E* − 07	1100.98
6	1.6960 *E* + 03	497.28	1.9869 *E* − 03	3.6173 *E* − 07	1093.18
8	1.5721 *E* + 03	490.15	1.8685 *E* − 03	3.4764 *E* − 07	1085.33
10	1.4556 *E* + 03	483.02	1.7556 *E* − 03	3.5353 *E* − 07	1077.40
12	1.3462 *E* + 03	475.90	1.6479 *E* − 03	3.4939 *E* − 07	1069.42
14	1.2436 *E* + 03	468.78	1.5455 *E* − 03	3.4522 *E* − 07	1061.38
16	1.1473 *E* + 03	461.66	1.4480 *E* − 03	3.4102 *E* − 07	1053.31
18	1.0575 *E* + 03	454.53	1.3553 *E* − 03	3.3679 *E* − 07	1045.14
20	9.7733 *E* + 02	447.42	1.2673 *E* − 03	3.3253 *E* − 07	1036.94
22	8.9459 *E* + 02	440.30	1.1836 *E* − 03	3.2825 *E* − 07	1028.64
24	8.2116 *E* + 02	433.18	1.1044 *E* − 03	3.2392 *E* − 07	1020.31
26	7.5270 *E* + 02	426.07	1.0292 *E* − 03	3.1958 *E* − 07	1011.88
28	6.8896 *E* + 02	418.95	9.5801 *E* − 04	3.1519 *E* − 07	1003.41
30	6.2966 *E* + 02	411.84	8.9070 *E* − 04	3.1078 *E* − 07	994.85
32	5.7457 *E* + 02	404.73	8.2704 *E* − 04	3.0633 *E* − 07	986.22
34	5.2347 *E* + 02	397.62	7.6695 *E* − 04	3.0185 *E* − 07	977.53
36	4.7611 *E* + 02	390.51	7.1029 *E* − 04	2.9734 *E* − 07	968.73
38	4.3262 *E* + 02	389.97	6.4640 *E* − 04	2.9700 *E* − 07	968.08
40	3.9311 *E* + 02	389.97	5.8728 *E* − 04	2.9700 *E* − 07	968.08
42	3.5722 *E* + 02	389.97	5.3366 *E* − 04	2.9700 *E* − 07	968.08
44	3.2477 *E* + 02	389.97	4.8494 *E* − 04	2.9700 *E* − 07	968.08
46	2.9477 *E* + 02	389.97	4.4068 *E* − 04	2.9700 *E* − 07	968.08
48	2.6806 *E* + 02	389.97	4.0046 *E* − 04	2.9700 *E* − 07	968.08
50	2.4360 *E* + 02	389.97	3.6393 *E* − 04	2.9700 *E* − 07	968.08
52	2.2138 *E* + 02	389.97	3.3072 *E* − 04	2.9700 *E* − 07	968.08
54	2.0119 *E* + 02	389.97	3.0056 *E* − 04	2.9700 *E* − 07	968.08
56	1.8288 *E* + 02	389.97	2.7315 *E* − 04	2.9700 *E* − 07	968.08
58	1.6618 *E* + 02	389.97	2.4824 *E* − 04	2.9700 *E* − 07	968.08
60	1.5103 *E* + 02	389.97	2.2561 *E* − 04	2.9700 *E* − 07	968.08
62	1.3726 *E* + 02	389.97	2.0505 *E* − 04	2.9700 *E* − 07	968.08
64	1.2475 *E* + 02	389.97	1.8637 *E* − 04	2.9700 *E* − 07	968.08
66	1.1339 *E* + 02	390.07	1.6934 *E* − 04	2.9706 *E* − 07	968.21
68	1.0307 *E* + 02	391.16	1.5351 *E* − 04	2.9775 *E* − 07	969.55
70	9.3725 *E* + 01	392.25	1.3920 *E* − 04	2.9845 *E* − 07	970.90

(*continues on next page*)